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# Testing for time variation in beta in India*

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**Abstract**

The beta of a stock is important in a variety of contexts, ranging from the cost of capital, asset pricing theory, to hedging using index derivatives. It is common to measure betas by estimating the market model using straight OLS in obtaining beta estimates. This assumes that betas are constant, despite strong economic arguments in favour of time–varying betas.

In this paper, we test for time–varying betas, in the context of a market model with GARCH errors, using the modified Kalman filter of Harvey et al. (1992). The null of beta constancy is rejected for 52% of stocks. This has significant implications for portfolio diversification and hedging.

JEL classification: G12

Keywords: Beta, CAPM, GARCH, Modified Kalman filter, Time–varying parameter model

# 1 Introduction

Stock betas are an important part of modern finance. They play a role in asset pricing theory, the calculation of 'abnormal returns' in event studies, the implementation of covariance-matrix estimators such as the single-index market model, estimation of cost of the capital, the calculation of hedge ratios on futures markets, etc.

The most common estimation strategy that is used by researchers and practitioners, for estimating the market model, is a straight OLS. However, there is a considerable literature focusing on the econometrics of beta estimation, exploring weaknesses of this estimation strategy and proposing improvements in various directions.

A key assumption that underlies OLS estimates for the market model is that the beta is constant. This is incompatible with a variety of economic arguments which predict time–varying betas. Beta is linked to leverage, which changes owing to changes in the stock price (Black 1976) or owing to decisions by the firm (Mandelker & Rhee 1984). There could be links between macro-economic variables and the firm beta, as illustrated in the work of Rosenberg & Guy (1976). When we interpret equity as a call option on the assets of the firm, the stock beta is related to the beta of the firm's assets through a factor which varies with the level of interest rates (Galai & Masulis 1976). Hence, changes in interest rate should generate changes in beta. Finally, beta is the covariance between the stock returns and index returns, scaled down by the variance of index returns. There is considerable evidence that index volatility is time–varying (Bollerslev et al. 1992) . This could generate time–variation in beta.

These arguments have led to a small literature on time–variation in betas. Broadly two approaches which have been adopted:

- One strategy is based on the Kalman filter, which allows us to model beta as a time–series process, and hence allows us to test for beta constancy (Bos & Newbold 1984). The standard Kalman filter is applicable to the problem of the market model with a time–varying beta, under the assumptions of normality and homoscedasticity of the error. A series of papers has conducted these tests in various countries: USA (Ohlson & Rosenberg 1982), Sweden (Wells 1996) etc. All these papers reject beta constancy.
- The second strategy uses a bivariate GARCH model (Bollerslev et al. 1988). Here, stock returns and index returns are assumed to follow GARCH processes,

and the time–varying covariance parameter is identified. This is directly relevant for the problem of computing hedge ratios for using index derivatives.

In this paper, we focus on tests based on Kalman filter. The traditional Kalman filter assumes that the market model residual is gaussian and homoscedastic. This is inconsistent with the considerable evidence which has accumulated about heteroscedasticity of financial returns (Bollerslev et al. 1988, Ng & Lilian 1991, Bollerslev et al. 1992). Harvey et al. (1992) derive the modified Kalman filter, which is quasi–optimal when errors show conditional heteroscedasticity. We apply this modified Kalman filter into testing for beta stationarity.

Our dataset uses daily returns on India's Bombay Stock Exchange and the NSE-50 index. We focus on 50 highly liquid stocks, over the time–period from 01/05/1996 to 30/03/2000. Our results may be summarized as follows:

- We find that the null hypothesis of beta constancy is rejected for 26 of these stocks.
- Of these beta proves to be a random coefficient for 12. For the remaining 14 stocks, beta proves to be mean–reverting.
- For the 26 stocks for which beta is time–varying, once we control or this time–variation, the average market–model $R^2$ goes up from 0.27 to 0.39.

These results show that India's equity market exhibits symptoms of significant time–variation in beta, as has been found with several other countries. As with the results elsewhere in the world, we find that beta estimation using OLS has important limitations. Conventional estimates of the market model $R^2$ are biased downwards. That is, conventional estimates understate the extent to which systematic risk is embedded in individual stocks and overstate the potential gains from diversification. These results also suggest that hedging using index derivatives would be significantly more effective if the hedge ratio is adjusted in the life of the hedging program.

The remainder of this paper is organised as follows. Section 2 discusses the importance of beta and its applications in modern finance. Section 3 discusses why the assumption of beta constancy may not be valid and the existing literature on time–varying beta. In Section 4 we state our model and describe the estimation methodology. Section 6 shows our results finally Section 7 concludes the paper with the main implications of our results.

## 2 Importance of beta

Beta has occupied center stage in both risk measurement and risk management since the concept was introduced by Markowitz (1959). It is one of the most widely used measures of risk among practitioners and financial economists. Beta has wide ranging applications in financial economics including testing of asset pricing theories, estimation of the cost of capital, evaluation of portfolio performance and calculation of hedge ratios for index derivatives, etc. Hence improvements in the measurement of beta would have useful ramifications for all these areas.

## 3 Problems with constant betas

Beta is commonly estimated using ordinary least square regression (OLS) of stock returns on market returns.[1] The OLS estimator is biased on stability of the market–model relationship over the estimation interval.

The stability of beta has been a matter of intense debate among researchers for the last three decades (Blume 1971, Vasicek 1973, Alexander & Chervani 1980, Brooks et al. 1998) . There are sound economic reasons that suggest that beta may be time–varying:

- Beta is linked to the leverage of the firm (Hamada 1972, Mandelker & Rhee 1984), hence changes in leverage would give a change in beta (Black 1976, Braun et al. 1995). Fluctuations in stock prices lead to changes in leverage, hence we may expect frequent changes in beta.
- Beta is a measure of risk of an asset vis-a-vis the market. Any news that will not affect market and stock returns uniformly will change beta of the stock. For instance, if an event increases variance of the market returns but leaves the variance of a security unchanged, then occurrence of that event will reduce the beta of that security (Rosenberg & Guy 1976).[2]
- Galai & Masulis (1976) interpret equity as a call option on the assets of the firm. They show that the beta of a stock is related to the beta of the firms assets through a factor that depends on the level of risk free interest rate.

[1] In the case of calculation of hedge ratios, returns on the hedging instrument (generally index futures) are used in place of market returns.

[2] Rosenberg & Guy (1976) show that beta of a security is the weighted average of its relative response coefficients for different events, each weighted by the proportion of total variance in market return due to that event. Beta will change either when the response coefficient changes or the relative variance of events change.

**Table 1** Models in the literature*

| Model | Study |
|---|---|
| Mean Reverting Model | Ohlson & Rosenberg (1982) |
| | Bos & Newbold (1984) |
| | Collins et al. (1987) |
| | Faff et al. (1992) |
| Random Coefficient Model | Fabozzi & Francis (1978) |
| | Simonds et al. (1986) |
| | Brooks et al. (1992) |
| Random Walk Model | Sunder (1980) |
| | LaMotte & McWhorter (1978) |

*See Wells (1996) for an excellent survey of literature.

Hence, changes in the risk free interest rates will generate time–variation in beta.

- There is considerable evidence that stock and index returns show time–varying second moments (Bollerslev et al. 1992). Since beta is the ratio of covariance between market and stock returns to the variance of market returns, time–variation in the second moments of returns can generate time–variation in beta.

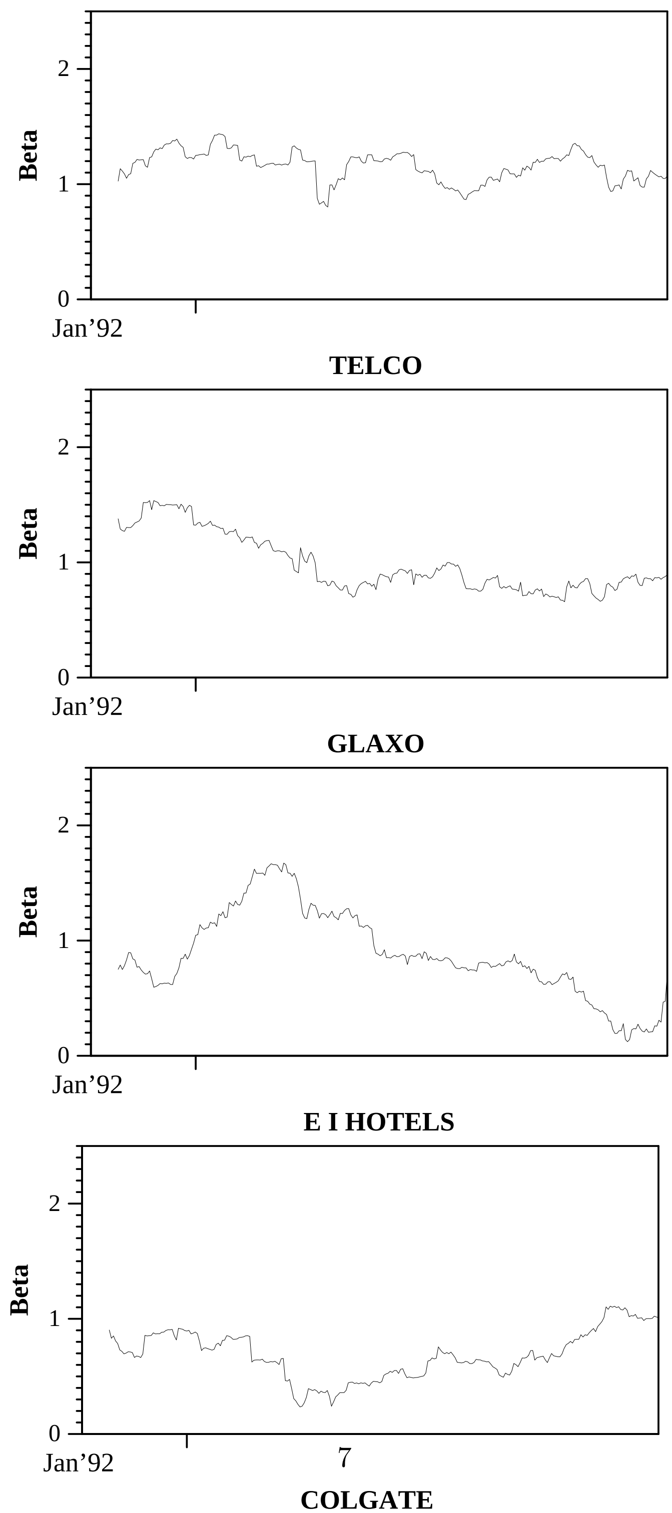
In the light of these economic arguments it is important to test the assumption of beta constancy.

For a preliminary exploration, we consider four stocks from the Indian stock market (Telco, Glaxo India, E I Hotels, Colgate). OLS betas of these stocks were estimated using a moving window of 50 weeks. The plots of these betas (Figure 1) do appear to display time–variation in beta. This encourages us to explore the question further.

### 3.1 Time–varying betas and the Kalman Filter

Economic arguments in favour of time–varying beta have led to the modeling of beta as a time–varying parameter. One way to model the time–variation in beta is to use bivariate GARCH models (Bollerslev et al. 1988). These models use time varying second moments of the market and index returns to obtain time varying betas. This approach has often been used in estimat-

**Figure 1** OLS betas using moving windows: Four examples



ing time varying hedge ratios (Kroner & Sultan 1993). Alternatively, beta can be estimated as a time–series process using the Kalman filter. This is particularly suitable for testing beta constancy, and this is the strategy we adopt. A number of studies have used Kalman filter to estimate one of the following time-series processes for beta (see Table 1).

1. Mean Reverting Beta (MREV).

$$\beta_t = \bar{\beta} + \phi(\beta_{t-1} - \bar{\beta}) + \eta_t \quad (1)$$

   with a "speed" parameter $\phi$. Here beta tends to go back towards $\bar{\beta}$

2. Random Coefficient Beta (RCM).

$$\beta_t = \bar{\beta} + \eta_t \quad (2)$$

   Here every realisation $\beta_t$ is an independent draw from a distribution with mean $\bar{\beta}$

3. Random Walk Beta (RWALK).

$$\beta_t = \beta_{t-1} + \eta_t \quad (3)$$

   Here $\beta_t$ reflects the cumulation of past draws of $\eta_t$.

# 4 Estimation methodology

The Kalman filter gives minimum mean square estimates (MMSE) of the state variable if the errors are normally distributed (see Harvey (1989)). This assumption of normality of errors in the market–model is incompatible with the abundant empirical evidence in favour of unconditional non–normality in financial time series. One important source of this non–normality is the phenomenon of volatility clustering which is often modeled using GARCH models (Bollerslev et al. 1992) . Previous papers that have employed the Kalman filter for estimating time–varying betas have ignored this non–normality of errors. Harvey et al. (1992) proposed a modified Kalman filter which can be used in presence of GARCH errors. We use this modified Kalman filter to estimate time–varying betas. The working of the modified Kalman filter is described in the next section.

### 4.1 Modified Kalman filter

The model that we estimate is

$$\begin{aligned} r_{it} &= \beta_{it} r_{mt} + \epsilon^*_{it} &(4)\\ \beta_{it} &= \bar{\beta}_i + \phi(\beta_{it-1} - \bar{\beta}_i) + \eta_{it} &(5) \end{aligned}$$

where

$$\epsilon^*_i \sim N(0, h_{it}) \qquad \eta_i \sim N(0, Q)$$

and

$$h_{it} = \omega_0 + \alpha_0 \epsilon^{*2}_{it-1} + \alpha_1 h_{it-1} \tag{6}$$

Here $r_{it}$ and $r_{mt}$ are the daily returns on the stock $i$ and the market respectively demeaned by the risk free interest rate.[3] Beta in this model is time dependent and is modeled as an AR(1) process described by Equation 5. Errors of the market model are conditionally normal and follow a GARCH(1,1) process given by Equation 6.

To use the modified Kalman filter of Harvey et al. (1992) we need to rewrite these equations as follows.

$$\begin{aligned} R_{it} &= \begin{pmatrix} R_{mt} & 1 \end{pmatrix} \begin{pmatrix} \beta_{it} \\ \epsilon^*_{it} \end{pmatrix} &(7)\\ \begin{pmatrix} \beta_{it} - \bar{\beta}_i \\ \epsilon^*_{it} \end{pmatrix} &= \begin{pmatrix} \phi & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \beta_{it-1} - \bar{\beta}_i \\ \epsilon^*_{it-1} \end{pmatrix} + \begin{pmatrix} \eta_{it} \\ \epsilon^*_{it} \end{pmatrix} &(8)\\ \mathrm{E}(\eta^*_{\mathrm{it}} \eta^{*'}_{\mathrm{it}}) &= \begin{pmatrix} Q & 0 \\ 0 & h_{it} \end{pmatrix} = Q^*_t &(9) \end{aligned}$$

or

$$\begin{aligned} R_{it} &= R^*_{mt} \beta^*_{it} \\ \beta^*_{it} &= \phi^* \beta^*_{it-1} + \eta^*_{it} \end{aligned}$$

Equation 7 and Equation 8 are the modified observation and state equations respectively. The Kalman filter consists of the following six equations.

[3] We use the 91 day treasury bill interest rate as the risk free interest rate. The risk free interest rate is assumed to remain constant over one month periods.

$$\begin{aligned}
\beta^*_{it|t-1} &= \phi^*\beta^*_{it-1|t-1} & (10)\\
P^*_{it|t-1} &= \phi^* P^*_{it-1|t-1}\phi^{*'} + Q^* & (11)\\
V^*_{it|t-1} &= R_{it} - R^*_{mt}\beta^*_{it|t-1} & (12)\\
F^*_{it|t-1} &= R^*_{mt}P^*_{it|t-1}R^{*'}_{mt} & (13)\\
\beta^*_{it|t} &= \beta^*_{it|t-1} + P^*_{it|t-1}R^{*'}_{mt}F^{*-1}_{t|t-1}V^*_{it|t-1} & (14)\\
P^*_{it|t} &= P^*_{it|t-1} - P^*_{it|t-1}R^{*'}_{mt}F^{*-1}_{it|t-1}R^*_{mt}P^*_{it|t-1} & (15)
\end{aligned}$$

where $P^*_i$, $F^*_i$ are the conditional variances of $\beta^*_i$ and $R^*_i$ respectively. Equations 10 and 11 are the two prediction equations and Equation 14 and 15 are the updating equations.

To apply equation 11 and 13 we need to calculate

$$h_{it} = \omega_0 + \alpha_0\epsilon^{*2}_{it-1} + \alpha_1 h_{it-1}$$

which is a function of the square of past unobservable errors. Thus the Kalman filter in the present form is not operable. The modified Kalman filter solves this problem by replacing $\epsilon^{*2}_{it-1}$ by its conditional expectation $\mathrm{E}(\epsilon^{*2}_{\mathrm{it-1}}|\psi_{\mathrm{t-1}})$. Thus the modified Kalman filter requires an approximation and hence is "quasi–optimal".

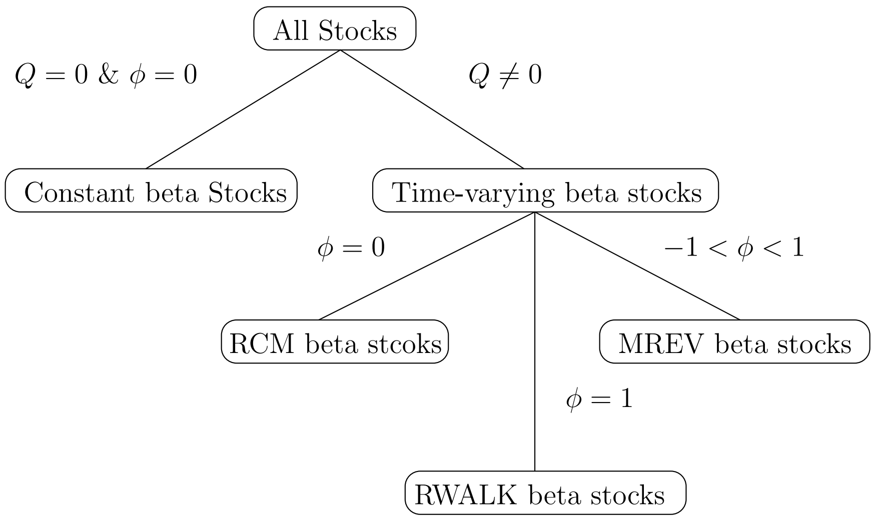
The parameters of the model are estimated by maximising the likelihood function:

$$L = \frac{1}{\sqrt{2\pi|F_{it|t-1}|}}\exp\left(-\frac{V'_{it|t-1}F^{-1}_{it|t-1}V_{it|t-1}}{2}\right) \qquad (16)$$

## 4.2 Testing for time–variation in beta

The model presented in the previous section describes how the time–variation in beta is modeled. The parameters to be estimated in this model are $\bar{\beta}$, $\phi$, $Q$, $\omega_0$, $\alpha_1$ and $\alpha_2$. This is done using the modified Kalman filter algorithm. To test the hypothesis of time–variation in beta, we need to test the null hypothesis of OLS beta ($H_0 : Q = 0$ & $\phi = 0$ ) versus the alternative of time varying beta ($H_1 : Q \neq 0$ & $\phi \neq 0$) in this model.

**Figure 2** Models for beta




### 4.3 Finding the correct model for time–varying betas

If we reject the hypothesis of beta constancy for a particular stock, the next stage is to find the appropriate model for beta variation. The three models that have been extensively used in the existing literature are the mean reverting model (Equation 1), the random coefficient model (Equation 2) and the random walk model (Equation 3).

Our testing procedure is summarised in Figure 4.3. To test the random coefficient model against the mean reverting model we impose the restriction $H_0 : \phi = 0$. The hypothesis is tested using an LR test at a 95% confidence level. For testing the random walk model we test the null hypothesis of $H_0 : \phi = 1, \bar{\beta} = 0$ against the mean reverting model using an LR test at a 95% confidence interval. If both the RCM and the RWALK models are rejected we accept the mean reverting model as the appropriate model for beta.

## 5 Data

Our dataset uses daily returns on the BSE[4] for 50 highly liquid stocks and the NSE-50 index. Our data covers the time period from 01/05/1996 to 30/03/2000 and was obtained from the PROWESS[5] database.

## 6 Results

### 6.1 Selecting models for time varying betas

The null hypothesis of constant beta was tested against the alternative of an MREV model. For 26 of the 50 stocks considered the hypothesis of constant beta was rejected at 95% confidence level (see Figure 6.1). Table 2 shows the stocks for which beta was found to be constant.

Once the stocks with constant beta were identified, the remaining 26 stocks were tested for the various time–varying beta models. The null hypothesis of RCM was tested against the alternative of MREV. Twelve stocks were found to have betas which behave as random coefficients (see Table 3).

Similarly the null hypothesis of beta following a random walk ($H_0 : \phi = 1$ & $\bar{\beta} = 0$) was tested against the alternative of a mean reverting model. We do not find support for the RWALK model for any stock.

The stocks which showed AR(1) parameters ($\phi$s) to be significantly different from zero and which could not be classified as RCM or RWALK beta stocks, were classified as MREV stocks. There were 14 stocks (Table 4) which fell in this category.

**Table 2** OLS beta stocks

| | $\bar{\beta}$ | $\omega$ | $\alpha_0$ | $\alpha_1$ |
|---|---|---|---|---|
| HCL | 1.15 | 7.40 | 0.088 | 0.84 |
| | (0.075) | (2.96) | (0.028) | (0.048) |
| ABB | 0.593 | 0.667 | 0.095 | 0.892 |
| | (0.039) | (0.240) | (0.022) | (0.023) |
| Bank of India | 0.926 | 15.658 | 0.280 | 0.410 |
| | (0.054) | (3.722) | (0.074) | (0.113) |
| BHEL | 0.986 | 1.264 | 0.049 | 0.923 |
| | 0.051 | 0.858 | 0.020 | 0.038 |
| Brittania | 0.498 | 2.649 | 0.141 | 0.792 |
| | (0.040) | (1.192) | (0.041) | (0.065) |
| Colgate Palmolive | 0.852 | 3.135 | 0.142 | 0.744 |
| | (0.036) | (1.421) | (0.040) | (0.084) |
| Dabur | 0.793 | 3.657 | 0.161 | 0.769 |
| | (0.051) | (1.345) | (0.044) | (0.062) |
| Dr. Reddy's Lab | 0.987 | 3.541 | 0.199 | 0.739 |
| | (0.044) | (1.143) | (0.044) | (0.054) |
| Glaxo | 0.753 | 0.596 | 0.071 | 0.915 |
| | (0.040) | (0.316) | (0.020) | (0.026) |
| Grasim | 0.578 | 0.173 | 0.023 | 0.976 |
| | (0.045) | (0.058) | (0.005) | (0.006) |
| HDFC Bank | 0.887 | 4.227 | 0.135 | 0.762 |
| | (0.042) | (1.097) | (0.029) | (0.047) |
| Hero Honda | 0.609 | 34.822 | 0.269 | 0.000 |
| | (0.049) | (2.127) | (0.054) | (0.001) |
| HLL | 0.640 | 0.723 | 0.095 | 0.869 |
| | (0.030) | (0.298) | (0.025) | (0.035) |
| Hindustan Petroleum | 0.594 | 1.343 | 0.143 | 0.830 |
| | (0.041) | (0.465) | (0.033) | (0.038) |
| Indian Petrochemicals | 1.038 | 2.480 | 0.096 | 0.857 |
| | (0.051) | (1.389) | (0.034) | (0.058) |
| Infosys Technology | 0.786 | 2.212 | 0.118 | 0.841 |
| | (0.048) | (1.041) | (0.031) | (0.048) |
| MTNL | 1.041 | 1.119 | 0.090 | 0.882 |
| | (0.043) | (0.637) | (0.038) | (0.051) |

Figures in the parenthesis are standard errors

| | $\bar{\beta}$ | $\omega$ | $\alpha_0$ | $\alpha_1$ |
|---|---|---|---|---|
| Reliance | 1.104 | 4.615 | 0.180 | 0.670 |
| | (0.039) | (1.625) | (0.045) | (0.088) |
| Satyam Computers | 1.168 | 2.515 | 0.098 | 0.873 |
| | (0.062) | (1.028) | (0.023) | (0.030) |
| Smithkline Beecham | 0.554 | 1.174 | 0.049 | 0.913 |
| | (0.041) | (0.425) | (0.013) | (0.023) |
| TISCO | 1.040 | 3.828 | 0.136 | 0.764 |
| | (0.042) | (1.209) | (0.033) | (0.055) |
| Tata Tea | 0.935 | 0.265 | 0.050 | 0.946 |
| | (0.0410) | (0.1360) | (0.0110) | (0.0120) |
| Zee Telefilms | 0.981 | 6.539 | 0.135 | 0.795 |
| | (0.0660) | (2.4200) | (0.0320) | (0.0510) |

Figures in the parenthesis are standard errors

**Figure 3** Results

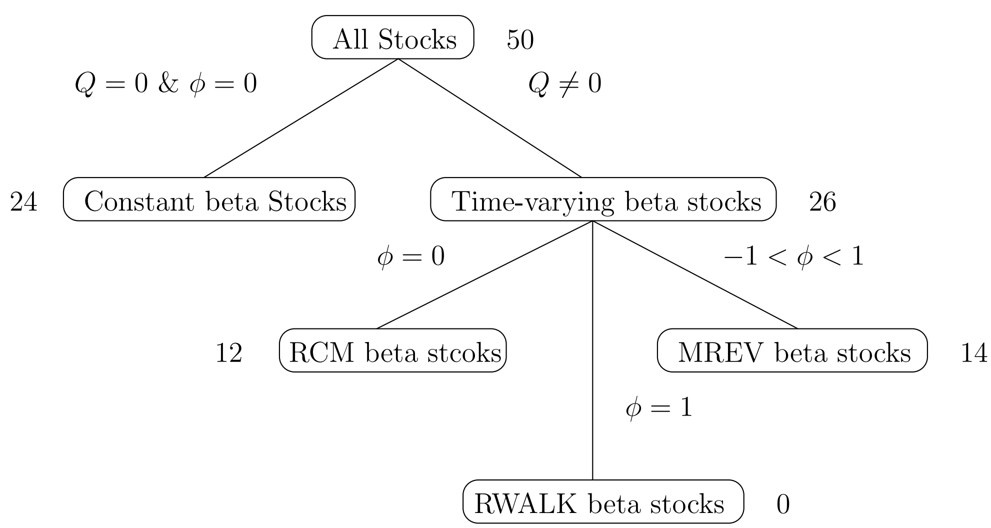


**Table 3** RCM beta stocks

| | $\bar{\beta}$ | $Q$ | $\omega$ | $\alpha_0$ | $\alpha_1$ |
|---|---|---|---|---|---|
| Castrol | 0.7028 | 0.1065 | 4.6899 | 0.3319 | 0.436 |
| | (0.03) | (0.04) | (0.97) | (0.07) | (0.08) |
| ICICI | 1.0275 | 0.2129 | 2.8293 | 0.144 | 0.8009 |
| | (0.05) | (0.10) | (1.02) | (0.04) | (0.05) |
| Larsen & Toubro | 1.143 | 0.1074 | 2.0418 | 0.1906 | 0.7516 |
| | (0.04) | (0.06) | (0.67) | (0.04) | (0.05) |
| Nestle | 0.692 | 0.1419 | 2.4073 | 0.2 | 0.7151 |
| | (0.04) | (0.05) | (0.71) | (0.04) | (0.05) |
| Novaratis | 0.4365 | 0.4716 | 9.6176 | 0.6794 | 0.1708 |
| | (0.05) | (0.09) | (1.73) | (0.11) | (0.07) |
| Procter & Gamble | 0.7661 | 0.138 | 5.1237 | 0.1643 | 0.675 |
| | (0.05) | (0.07) | (1.29) | (0.04) | (0.06) |
| Tata Chemicals | 0.882 | 0.1832 | 0.7795 | 0.1252 | 0.8617 |
| | (0.04) | (0.06) | (0.26) | (0.03) | (0.03) |
| TELCO | 1.0722 | 0.1958 | 0.6818 | 0.117 | 0.875 |
| | (0.05) | (0.08) | (0.35) | (0.03) | (0.03) |
| Corporation Bank | 1.0845 | 0.2376 | 22.9784 | 0.3488 | 0.2328 |
| | (0.07) | (0.14) | (6.01) | (0.11) | (0.17) |
| Hoechst Mario | 0.5854 | 0.1611 | 17.6055 | 0.287 | 0.4217 |
| | (0.06) | (0.10) | (4.16) | (0.06) | (0.10) |
| IDBI | 0.7791 | 0.3844 | 4.9786 | 0.3588 | 0.5155 |
| | (0.05) | (0.09) | (1.29) | (0.07) | (0.08) |
| Punjab Tractors | 0.5843 | 0.1777 | 5.5763 | 0.2352 | 0.5855 |
| | (0.05) | (0.07) | (1.70) | (0.05) | (0.09) |
| TVS Suzuki | 0.5918 | 0.3153 | 4.5865 | 0.2176 | 0.6586 |
| | (0.05) | (0.08) | (1.02) | (0.05) | (0.05) |

Figures in the parenthesis are standard errors

**Table 4** MREV beta stocks

| | $\bar{\beta}$ | $\phi$ | $Q$ | $\omega$ | $\alpha_0$ | $\alpha_1$ |
|---|---|---|---|---|---|---|
| Asian Paints | 0.487 | 0.8048 | 0.0391 | 1.1383 | 0.1879 | 0.7828 |
| | (0.05) | (0.07) | (0.02) | (0.32) | (0.04) | (0.04) |
| Cipla | 0.5588 | 0.5717 | 0.2058 | 1.7752 | 0.1676 | 0.7971 |
| | (0.06) | (0.20) | (0.13) | (0.52) | (0.03) | (0.03) |
| Gujarat Ambuja | 0.8628 | 0.7024 | 0.1006 | 0.6552 | 0.0698 | 0.9147 |
| | (0.06) | (0.14) | (0.06) | (0.30) | (0.02) | (0.02) |
| Hindalco | 0.6482 | 0.7444 | 0.0989 | 0.703 | 0.1557 | 0.8322 |
| | (0.06) | (0.10) | (0.04) | (0.26) | (0.04) | (0.04) |
| Indian Hotels | 0.5477 | 0.6927 | 0.0933 | 1.9294 | 0.2711 | 0.6772 |
| | (0.05) | (0.14) | (0.05) | (0.70) | (0.08) | (0.08) |
| Kochi Refineries | 0.7689 | 0.6866 | 0.2142 | 10 | 0.1261 | 0.6333 |
| | (0.08) | (0.11) | (0.10) | (2.60) | (0.04) | (0.04) |
| NIIT | 0.8159 | 0.6533 | 0.1833 | 10 | 0.3326 | 0.4866 |
| | (0.07) | (0.15) | (0.09) | (3.20) | (0.07) | (0.05) |
| Oriental Bank | 0.8614 | 0.9326 | 0.0199 | 9.054 | 0.248 | 0.5098 |
| | (0.08) | (0.03) | (0.01) | (2.35) | (0.06) | (0.10) |
| Reckitt & Colman | 0.3761 | 0.9881 | 0.0037 | 0.5114 | 0.1984 | 0.8016 |
| | (0.20) | (0.01) | (0.00) | (0.18) | (0.04) | (0.04) |
| Cadbury | 0.5652 | 0.8879 | 0.033 | 1.6675 | 0.1243 | 0.831 |
| | (0.07) | (0.05) | (0.02) | (0.57) | (0.03) | (0.04) |
| Madras Cements | 0.7064 | 0.4042 | 0.2615 | 4.4991 | 0.323 | 0.5782 |
| | (0.06) | (0.18) | (0.09) | (1.33) | (0.06) | (0.07) |
| HLL | 0.6411 | 0.8324 | 0.0205 | 1.0314 | 0.1263 | 0.8186 |
| | (0.04) | (0.09) | (0.01) | (0.32) | (0.03) | (0.04) |
| Indian Petroleum | 1.0765 | 0.857 | 0.0471 | 3.1163 | 0.1292 | 0.8074 |
| | (0.07) | (0.07) | (0.03) | (1.27) | (0.04) | (0.06) |

Figures in the parenthesis are standard errors

| | $\bar{\beta}$ | $\phi$ | $Q$ | $\omega$ | $\alpha_0$ | $\alpha_1$ |
|---|---|---|---|---|---|---|
| SAIL | 1.1675 | 0.7466 | 0.3353 | 2.1721 | 0.0445 | 0.9334 |
| | (0.11) | (0.16) | (0.24) | (0.99) | (0.02) | (0.03) |
| Ranbaxy Labs | 0.3761 | 0.9881 | 0.0037 | 0.5114 | 0.1984 | 0.8016 |
| | (0.20) | (0.01) | (0.00) | (0.18) | (0.04) | (0.04) |
| Reliance Petro | 0.7648 | 0.8551 | 0.0744 | 4.0229 | 0.2098 | 0.6766 |
| | (0.08) | (0.06) | (0.03) | (1.41) | (0.06) | (0.09) |
| Brittania | 0.5007 | 0.9818 | 0.0016 | 3.4505 | 0.1575 | 0.7487 |
| | (0.09) | (0.01) | (0.002) | (1.69) | (0.05) | (0.09) |
| Hindustan Petro | 0.6325 | 0.947 | 0.0064 | 1.5568 | 0.1636 | 0.8045 |
| | (0.07) | (0.05) | (0.01) | (0.52) | (0.04) | (0.04) |
| Apollo Tyres | 0.9967 | 0.9625 | 0.0029 | 1.5969 | 0.1016 | 0.877 |
| | (0.08) | (0.02) | (0.004) | (0.62) | (0.03) | (0.03) |
| Bharat Petroleum | 0.6085 | 0.9169 | 0.005 | 3.9565 | 0.1752 | 0.7535 |
| | (0.06) | (0.05) | (0.01) | (1.22) | (0.04) | (0.06) |
| Crompton Greaves | 1.1971 | 0.9895 | 0 | 6.3607 | 0.1309 | 0.8017 |
| | (0.09) | (0.00) | . | (2.16) | (0.03) | (0.04) |

Figures in the parenthesis are standard errors

**Table 5** Performance comparison

| | Assuming Constant $\beta$ | | Assuming time–varying $\beta$ | |
|---|---|---|---|---|
| | $R^2$ | Var($\epsilon$) | $R^2$ | Var($\epsilon$) |
| Asian Paints | 0.190 | 25.40 | 0.302 | 22.12 |
| Bajaj Auto | 0.333 | 27.09 | 0.478 | 21.42 |
| BSES | 0.363 | 36.50 | 0.443 | 32.28 |
| Castrol | 0.340 | 19.57 | 0.451 | 16.48 |
| Cipla | 0.139 | 46.19 | 0.344 | 35.56 |
| Gujarat Ambuja | 0.293 | 37.93 | 0.409 | 32.06 |
| Hindalco | 0.228 | 31.31 | 0.375 | 25.65 |
| HDFC | 0.218 | 33.66 | 0.323 | 29.46 |
| ICICI | 0.271 | 53.12 | 0.365 | 46.83 |
| Indian Hotels | 0.219 | 29.13 | 0.372 | 23.66 |
| ITC | 0.412 | 32.78 | 0.518 | 27.16 |
| Kochi Refineries | 0.203 | 48.56 | 0.412 | 36.19 |
| Larsen & Toubro | 0.466 | 29.92 | 0.497 | 28.44 |
| Mahindra & Mahindra | 0.220 | 48.25 | 0.256 | 46.52 |
| Nestle | 0.266 | 27.74 | 0.377 | 23.79 |
| NIIT | 0.220 | 53.68 | 0.351 | 45.14 |
| Novartis | 0.116 | 42.15 | 0.392 | 29.31 |
| Oriental Bank | 0.299 | 37.94 | 0.371 | 34.38 |
| Procter & Gamble | 0.286 | 32.91 | 0.386 | 28.61 |
| Ranbaxy | 0.177 | 35.76 | 0.328 | 29.52 |
| Reckitt & Colman | 0.251 | 32.38 | 0.378 | 27.15 |
| Reliance Petro | 0.270 | 35.47 | 0.416 | 28.70 |
| SBI | 0.462 | 31.46 | 0.518 | 28.50 |
| Tata Chem | 0.299 | 36.49 | 0.352 | 34.11 |
| TELCO | 0.344 | 45.14 | 0.371 | 43.79 |
| Tata Power | 0.281 | 34.34 | 0.414 | 28.28 |
| Average | 0.276 | 36.34 | 0.392 | 30.97 |

### 6.2 Do time–varying betas perform better?

To measure the improvement in fit over the conventional OLS beta market–model, we use two measures: the coefficients of determination ($R^2$) and the variances of the errors. The comparison of coefficients of determination and variances has to be done carefully. The Kalman filter exercise gives the one step ahead prediction residuals and the residuals formed from the updated states. Which of these residuals should be used for calculating the $R^2$s and variances? The coefficient of determination calculated using one step ahead prediction errors will be at a disadvantage compared to the OLS $R^2$ as the latter is estimated using the entire dataset and hence is using more information than the former. The same problem is found with variances also. Since our objective is to measure gains in accuracy over the OLS methodology, we use residuals from updated states to calculate $R^2$ and market model error variance.

Our results show significant gains in a accuracy (see Table 5) in terms of a higher $R^2$ and lower variance when beta is allowed to follow an appropriate[6] time–varying model.

## 7 Conclusion

Our results show that the Indian stock market exhibits symptoms of time–varying betas. This is in line with previous studies which have found evidence of beta variation in various other countries. Our results show a tendency for beta to be mean reverting and show little evidence of beta as a random walk process.

The strong evidence in favour of time–varying betas highlights the limitations of OLS betas. The superior performance of time–varying betas as opposed to OLS betas can be judged by the improvement in the market model $R^2$ and reduction in the variance of errors. The average $R^2$ over the sample of stocks studied increased from 0.27 to 0.39 when betas were allowed to be time–varying.

[4] The Bombay Stock Exchange is the second largest stock exchange in India.

[5] The PROWESS database is maintained by The Centre for Monitoring Indian Economy.

[6] By 'appropriate' we mean the model selected for that stock among the four models studied.

One of the contributions of this work over the existing literature is that we account for time–variation in second moments of the market model errors while estimating these models using a Kalman filter algorithm. All the previous works of this kind, to best of our knowledge, have ignored volatility clustering. We also measure the magnitude of reduction in the variance of market model errors that can be achieved if betas are allowed to vary. This gives us a useful metric for comparing time–varying beta models with the constant beta model. Finally, work on testing for time–variation in beta, so far, has been done for developed countries. We add to this body of evidence with results for a new market.

Our results have important implications for portfolio diversification and hedging strategies. One of the implications is that conventional OLS estimates of beta understate the extent of systematic risk embedded in a stock, thus overstating the potential gains from diversification. A second implication of our results is that a dynamic hedging strategy, in which the hedge ratios are frequently adjusted in light of the new information, will perform better compared to a static strategy where the hedge ratio is chosen at the beginning of the investment horizon.

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