

The contributions are non-stochastic. Suppose they are  $C_i$ .  
 Suppose the initial wealth starts at 1.

$$\begin{aligned} W_0 &= 1 \\ W_1 &= C_1 + W_0(1 + r_1) \\ W_2 &= C_2 + (1 + r_2)(C_1 + W_0(1 + r_1)) \\ W_3 &= C_3 + (1 + r_3)(C_2 + (1 + r_2)(C_1 + W_0(1 + r_1))) \end{aligned}$$

The expression for  $W_3$  simplifies to:

$$W_3 = C_3 + (1 + r_3)C_2 + (1 + r_3)(1 + r_2)C_1 + (1 + r_3)(1 + r_2)(1 + r_1)$$

which makes one notice a pattern!

$W_3$  is the dot product between the two vectors:

$$\begin{array}{r} 1 \\ (1 + r_3) \\ (1 + r_3)(1 + r_2) \\ (1 + r_3)(1 + r_2)(1 + r_1) \end{array} \quad \begin{array}{l} C_3 \\ C_2 \\ C_1 \\ 1 \end{array}$$

Hmm, this can be done elegantly in R:

```
c(1, cumprod(rev(r))) %*% c(rev(contributions), 1)
```

Note that the right hand side of the dot product (the vector on the right in the table) doesn't change across each simulation, so it can be precomputed.

Further, whether you do `rev(r)` or not doesn't matter :-). So you can pare down to:

```
precomputed <- c(rev(contributions), 1)
c(1, cumprod(r)) %*% precomputed
```